DYNAMICS OF NONSTATIONARY HEAT-TRANSFER PROCESSES IN MIXING CHAMBERS

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The transient processes in mixing chambers associated with perturbations of the inlet temperature of the heat-transfer agent are examined considering the heat exchange with the structural elements of the chamber.

In investigating the nonstationary operating regimes of equipment such as mixing chambers it is necessary to consider the heat transfer between the heat-transfer agent and the structural elements of the chamber observed in connection with perturbations of the inlet temperature. Nuclear reactor mixing tanks and heat-exchanger headers are examples of such chambers. In them the flow of heat-transfer agent is such that ideal mixing can be assumed, throughout the chamber. It was shown experimentally [1] that this assumption, widely used in investigating transient processes in equipment with mixing chambers, is quite realistic. The heat transfer with the structural elements of the chamber has been neglected, although in a number of cases it should definitely be taken into account.

Accurately enough for practical purposes the structural elements exposed to the heat-transfer agent (chamber walls, various rods and plates) can be represented by cylinders and plates, with one dimension (thickness, height, radius) smaller than the others. Accordingly we confine ourselves to a one-dimensional description of the temperature field. Of course, it is possible to take the multidimensionality of the temperature field in the structural elements into account; however, this leads to considerable complication of the final expressions and complicates the practical application of the solution without much improvement in accuracy. The problem is also complicated if the structural elements consist of several layers with different thermophysical properties. In this case an approximate single-layer model can be used by introducing "effective" thermophysical constants.

The heat-transfer process is described by the following system, which includes the heat conduction equations for all the elements, the heat balance equation, the initial conditions, the boundary conditions of the third kind at the exposed surfaces of the elements, and the conditions of symmetry at the coordinate origin (this is taken on the insulated surface for elements exposed on one side, and in the center for elements exposed on both sides):

$$\frac{\partial t_j}{\partial \tau} = a_j \nabla^2 t_j \quad (j = 1, \ldots, k), \tag{1}$$

$$\nabla^2 t_j = \frac{\partial^2 t_j}{\partial x_j^2}, \text{ for flat elements}$$

$$\nabla^2 t_j = \frac{\partial^2 t_j}{\partial x_j^2} + \frac{1}{x_j}\frac{\partial t_j}{\partial x_j}, \text{ for cylindrical elements.}$$

The initial conditions are

$$\tau = 0, \quad t_1 = t_2 = \ldots = t_k = 0, \quad \theta = 0. \tag{2}$$

The boundary conditions are

$$x_j = 0, \quad \frac{\partial t_j}{\partial x_j} = 0, \tag{3}$$

$$x_j = \delta_j, \quad \lambda_j \frac{\partial t_j}{\partial x_j} = \alpha_j(\theta - t_j), \tag{4}$$

$$GC_\tau \theta_0 = GC_\tau \theta + V_0 C_\tau \frac{d\theta}{d\tau} + \sum_{j=1}^{k} F_j \lambda_j \left.\frac{\partial t_j}{\partial x_j}\right|_{x_j=\delta_j}. \tag{5}$$

The Laplace transforms are

$$L\left[t_j(x, \mathrm{Fo})\right] = T_j(x, s), \quad L\left[\theta(\mathrm{Fo})\right] = U(s),$$

$$L\left[\theta_0(\mathrm{Fo})\right] = U_0(s), \quad \mathrm{Fo} = \frac{a_1\tau}{\delta_1^2}.$$

Solving system (1)–(5) in the transforms, we find the transfer function

$$w_{U_0U} = \frac{1}{1 - \mu^2 \mathrm{Fo}^* - \sum\limits_{j=1}^{k} \dfrac{P_j \mu\varepsilon_j \psi_1(\mu\varepsilon_j)}{\psi_2(\mu\varepsilon_j) - \dfrac{\mu\varepsilon_j}{\mathrm{Bi}_j}\psi_1(\mu\varepsilon_j)}},$$

$$s = -\mu^2 = i\omega\frac{\delta_1^2}{a_1},$$

$$\mathrm{Fo}^* = \frac{V_0}{G}\frac{a_1}{\delta_1^2}, \quad P_j = \frac{F_j\lambda_j}{GC_\tau\delta_j},$$

$$\mathrm{Bi}_j = \frac{\alpha_j\delta_j}{\lambda_j}, \quad \varepsilon_j = \frac{\delta_j}{\delta_1}\sqrt{\frac{a_1}{a_j}},$$

$$\psi_1(\mu\varepsilon_j) = \sin(\mu\varepsilon_j),$$

$$\psi_2(\mu\varepsilon_j) = \cos(\mu\varepsilon_j) \text{ for flat elements, and}$$

$$\psi_1(\mu\varepsilon_j) = I_1(\mu\varepsilon_j),$$

$$\psi_2(\mu\varepsilon_j) = I_0(\mu\varepsilon_j) \text{ for cylindrical elements.} \tag{6}$$

The inverse transform of the transfer function

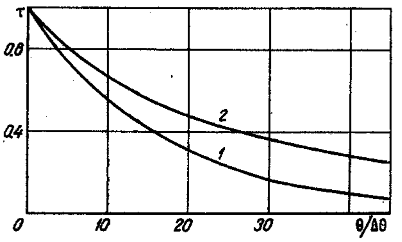
$$\Omega = -\sum_{n=0}^{\infty} 2\mu_n \frac{\prod\limits_{j=1}^{k} M_j(\mu_n)\exp(-\mu_n^2 \mathrm{Fo})}{\Phi'(\mu_n)}, \tag{7}$$

$$\Phi(\mu) = (1-\mu^2 \mathrm{Fo}^*) \prod_{j=1}^{k} M_j(\mu) - $$
$$-\sum_{\rho=1}^{k} \prod_{j=1}^{\rho-1} [M_j(\mu)]\, N_\rho(\mu) \prod_{j=\rho+1}^{k} [M_j(\mu)], \tag{8}$$
$$\Phi'(\mu) = \frac{d\Phi}{d\mu},$$
$$M_j(\mu) = \psi_2(\mu\varepsilon_j) - \frac{\mu\varepsilon_j}{\mathrm{Bi}_j}\psi_1(\mu\varepsilon_j), \tag{9}$$
$$N_j(\mu) = P_j \mu\varepsilon_j \psi_1(\mu\varepsilon_j), \tag{10}$$

where μ_n are the roots of

$$\Phi(\mu) = 0. \tag{11}$$

The temperature of the heat-transfer agent at the chamber outlet for an arbitrary perturbation of the inlet temperature is



Sodium temperature at chamber outlet as a function of time (τ, sec): curve 1 was calculated without allowance for heat exchange with the structural elements; and curve 2 was calculated with allowance for heat exchange.

$$\theta = \int_0^{\mathrm{Fo}} \Omega(\mathrm{Fo}-\mathrm{Fo}')\,\theta_0(\mathrm{Fo}')\,d\mathrm{Fo}' = $$
$$= \int_0^{\mathrm{Fo}} \Omega(\mathrm{Fo}')\,\theta_0(\mathrm{Fo}-\mathrm{Fo}')\,d\mathrm{Fo}'.$$

We will consider some typical perturbations.

(1) Exponential perturbation $\theta_0 = \Delta\theta[1 - \exp(-m\,\mathrm{Fo})]$ for which

$$\frac{\theta-\theta_0}{\Delta\theta} = \sum_{n=0}^{\infty} \frac{2\mu_n m}{\mu_n^2(\mu_n^2 - m)\,\Phi'(\mu_n)} \times$$
$$\times \prod_{j=1}^{k} [M_j(\mu_n)]\,[\exp(-m\,\mathrm{Fo}) - \exp(-\mu_n^2 \mathrm{Fo})]. \tag{12}$$

(2) An instantaneous jump $\Delta\theta$ in inlet temperature for which

$$\frac{\theta}{\Delta\theta} = 1 + \sum_{n=0}^{\infty} \frac{2\exp(-\mu_n^2 \mathrm{Fo})}{\mu_n \Phi'(\mu_n)} \prod_{j=1}^{k} M_j(\mu_n). \tag{13}$$

(3) A linear variation $\Delta\theta\Delta\mathrm{Fo}$ in inlet temperature in time $\Delta\mathrm{Fo}$ for which $\mathrm{Fo} \le \Delta\mathrm{Fo}$ gives

$$\frac{\theta-\theta_0}{\Delta\theta} = \sum_{n=0}^{\infty} \frac{2\,[1-\exp(-\mu_n^2 \mathrm{Fo})]}{\mu_n^3 \Phi'(\mu_n)} \prod_{j=1}^{k} M_j(\mu_n), \tag{14}$$

and for which $\mathrm{Fo} > \Delta\mathrm{Fo}$ gives

$$\frac{\theta-\theta_0}{\Delta\theta} = \sum_{n=0}^{\infty} \frac{2\exp(-\mu_n^2 \mathrm{Fo})}{\mu_n^3 \Phi'(\mu_n)} \prod_{j=1}^{k} M_j(\mu_n) \times$$
$$\times [\exp(\mu_n^2 \Delta\mathrm{Fo}) - 1]. \tag{15}$$

(4) Finally, the harmonic perturbation $\theta_0 = \Delta\theta \times \sin(\omega_1 \mathrm{Fo})$ for which

$$\frac{\theta}{\Delta\theta} = \sum_{n=0}^{\infty} 2\mu_n \frac{\prod_{j=1}^{k} M_j(\mu_n)}{\Phi'(\mu_n)} \times$$
$$\times \left\{ \frac{\omega_1}{\mu_n^4 + \omega_1^2} [\cos(\omega_1 \mathrm{Fo}) - \exp(-\mu_n^2 \mathrm{Fo})] - \right.$$
$$\left. - \frac{\sin(\omega_1 \mathrm{Fo})}{\mu_n^4 + \omega_1^2} \mu_n^2 \right\}.$$

From the latter expression we find the amplitude response of the system

$$A(\omega) = \left\{ \left[\sum_{n=0}^{\infty} \frac{2\mu_n^3 \prod_{j=1}^{k} M_j(\mu_n)}{\Phi'(\mu_n)(\mu_n^4 + \omega_1^2)} \right]^2 + \right.$$
$$\left. + \left[\sum_{n=0}^{\infty} \frac{2\mu_n \omega_1 \prod_{j=1}^{k} M_j(\mu_n)}{\Phi'(\mu_n)(\mu_n^4 + \omega_1^2)} \right]^2 \right\}^{1/2} \tag{16}$$

and the phase response

$$\varphi(\omega) = \arcsin \frac{-\sum_{n=0}^{\infty} \frac{2\mu_n \omega_1 \prod_{j=1}^{k} M_j(\mu_n)}{\Phi'(\mu_n)(\mu_n^4 + \omega_1^2)}}{A(\omega)}. \tag{17}$$

Obviously, the gain of the system is equal to 1, i. e.,

$$-\sum_{n=0}^{\infty} \frac{2 \prod_{j=1}^{k} M_j(\mu_n)}{\mu_n \Phi'(\mu_n)} = 1.$$

The dimensionless ω_1 and dimensional ω frequencies are related by

$$\omega = \omega_1 \frac{a_1}{\delta_1^2}.$$

Series (16) and (17) converge rapidly, and in accordance with these expressions the system can easily be approximated by an element with a rational transfer function.

Such an approximation is necessary in electrical analog simulation of a mixing chamber forming part of a chain with other linear elements (for example, heat exchange equipment, pipelines, etc.)

The figure presents the results of a calculation of the temperature of liquid sodium at a 0.9 m³ mixing chamber outlet following an instantaneous change of temperature at the inlet. The flow rate of sodium through the chamber is 0.054 m^3/sec; in flowing through the chamber the sodium comes in contact with the walls, of 0.068 m thickness and total area 5.2 m^2, a plate 0.1 m thick and 1.8 m^2 in area, and a cylindrical rod 0.05 m in diameter and 0.08 m^2 in area. All these elements are made of stainless steel. The calculation shows that heat transfer exerts significant influence on the transient process.

NOTATION

t is the temperature of the structural element; θ is the temperature of the heat-transfer agent; θ_0 is the temperature of the heat-transfer agent at chamber inlet; τ is the time; x is a space coordinate; δ is the thickness of a flat or the radius of a cylindrical element; G is the flow rate of the heat-transfer agent through chamber; V_0 is the mass of heat-transfer agent in chamber; F is the heat-transfer surface; and ω is the frequency. Subscripts: T denotes the heat-transfer agent; j denotes the j-th element; and the remaining notation is standard.

REFERENCE

1. L. Koch, V. Simmens, and M. Levenson, et al., "Nuclear power reactor without a moderator (EBR-II)," Selected papers presented by foreign scientists at the Second International Conference on the Peaceful Uses of Atomic Energy, Vol. IV [Russian translation], izd. GUIAE, Moscow, 1959.

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